Data Structures

(Qualify Exam, 2010 Spring)

1. (8%) Assume a tree T has m edges in total, where m is an even number. Removing one edge from T yields two disjoint trees T_1 and T_2. Given that the number of vertices in T_1 is equal to the number of edges in T_2. Determine the number of edges in T_1 and the number of vertices in T_2.

2. (12%) Prove that a graph G is bipartite if and only if it contains no cycles of odd length.

3. (15%) This question contains three sub-questions as follows.

(a) (5%) What is a binary search tree?

(b) (5%) What is a max-heap?

(b) (5%) Please describe the Prim's algorithm.

4. (7%) We said that `HeapSort` is inherently unstable. Please explain why and give an example.

5. (16%)

(a) Show the contents of the array given below

43	7	10	23	18	4	19	5	66	14
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]

after the `4th` iteration of

(1) Bubble Sort

(2) Selection Sort

(3) Insertion Sort

(b) Show how the values in the array in (a) would have to be rearranged to satisfy the heap property.

6. (5%) The element being searched for is not in an array of 100 elements. What is the `maximun` number of comparisons needed in a sequential search to determine that the element is not there? Please prove your answer.

(a) if the elements are completely unsorted?

(b) if the elements are sorted from smallest to largest?

(c) if the elements are sorted from largest to smallest?

7. (5%) The element being searched for is in an array of 100 elements. What is the average number of comparisons needed in a sequential search to determine the position of the element? Please prove your answer.

(3) if the elements are completely unsorted?

(b) if the elements are sorted from smallest to largest?

(c) if the elements are sorted from largest to smallest?

8. (6%) What is a queue? (6%) What is a stack?

9. (10%) Can you implement one stack by two queues? Please prove your answer.

10. (10%) Can you implement one queue by two stacks? Please prove your answer.